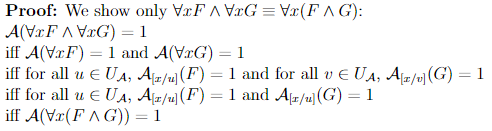
Additional Exercises 14

⋀⋁≡ㄱ





From F ⋁ Q to ㄱ(ㄱQ ⋀ ㄱF):

If 𝓐(F ⋁ Q) = 1 **then** 𝓐(F) = 1 or 𝓐(Q) = 1

If 𝓐(F) = 1 or 𝓐(Q) = 1 **then** 𝓐(ㄱF) = 0 or 𝓐(ㄱQ) = 0

If 𝓐(ㄱF) = 0 or 𝓐(ㄱQ) = 0 **then** 𝓐(ㄱF ⋀ ㄱQ) = 0

If 𝓐(ㄱF ⋀ ㄱQ) = 0 **then** 𝓐(ㄱ(ㄱF ⋀ ㄱQ)) = 1

From ㄱ(ㄱQ ⋀ ㄱF) to F ⋁ Q

If 𝓐(ㄱ(ㄱF ⋀ ㄱQ)) = 1 **then** 𝓐(ㄱF ⋀ ㄱQ) = 0

If 𝓐(ㄱF ⋀ ㄱQ) = 0 **then** 𝓐(ㄱF) = 0 or 𝓐(ㄱQ) = 0

If 𝓐(ㄱF) = 0 or 𝓐(ㄱQ) = 0 **then** 𝓐(F) = 1 or 𝓐(Q) = 1

If 𝓐(F) = 1 or 𝓐(Q) = 1 **then** 𝓐(F ⋁ Q) = 1



∀x∃y( P(x) ⋀ Q(y) ) ≢ ∃u∃v( P(v) ⋀ Q(u) )

UA = { 0, 1 }

PA = { 1 }

QA = { 1 }

Right side:

𝓐( ∃u∃v( P(v) ⋀ Q(u) ) ) = 1

Iff 𝓐( ∃u∃v( P(v) ) = 1 and 𝓐( ∃u∃v( Q(u) ) = 1

There exists a ∈ UA and b ∈ UA such that b ∈ (P(v))𝓐[u/a, v/b]

and there exists a ∈ UA and b ∈ UA such that a ∈ (Q(u))𝓐[u/a, v/b]

Simplification step - since there is no u in P(v), and there is no v in Q(u):

There exists b ∈ UA such that b ∈ (P(v))𝓐[v/b]

and there exists a ∈ UA such that a ∈ (Q(u))𝓐[u/a]

Since 1 ∈ (P(v))𝓐[v/1] and 1 ∈ (Q(u))𝓐[u/1],

Then 𝓐 is a model for ∃u∃v( P(v) ⋀ Q(u) ).

Left side:

𝓐( ∀x∃y( P(x) ⋀ Q(y) ) ) = 1

Iff 𝓐( ∀x∃y( P(x) ) = 1 and 𝓐( ∀x∃y( Q(y) ) = 1

For all b ∈ UA such that b ∈ (P(x))𝓐[x/b]

there exists a ∈ UA such that a ∈ (Q(y))𝓐[y/a]

But since 0 ∉ (P(v))𝓐[v/0]

Then 𝓐 is not a model for ∀x∃y(P(x) ⋀ Q(y)).

Since A is a model for one of the formulas but not the other, they are not equivalent.

*Note: Equivalence only if they have exactly the same models*



∀y∃z( ㄱQ(y,z) ⋀ ∀xP(x) ) ≡ ∀x( P(x) ⋀ ㄱ∃y∀zQ(y, z) )

≡ ∀x( P(x) ⋀ ∀y∃zㄱQ(y, z) )

≡ ∀xP(x) ⋀ ∀y∃zㄱQ(y, z)

≡ ∀y∃z( ∀xP(x) ⋀ ㄱQ(y,z) )

≡ ∀y∃z( ㄱQ(y,z) ⋀ ∀xP(x) )]



⋀⋁≡ ㄱ ∀∃

Only substitute if variable is free within its scope.



(∀xP(x, x) ⋀ ∃y∀zR(y, y, z))[x/a][a/c][x/d] = (∀xP(x, x) ⋀ ∃y∀zR(y, y, z))



((∃y∀z∀xQ(y, z))[z/a][a/b] ⋀ ∀zR(z, z, x))[y/x][x/b] = ((∃y∀z∀xQ(y, z)) ⋀ ∀zR(z, z, b)

Skip 61:







